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The Convergent Flow of Polymeric Liquids

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SUMMARY

An analysis is performed of the changes of the hydrodinamic field which express theirself in a formation of a birefringent inlet stream on flowing of the elastic liquid in an orifice. It is shown that the deformation of pure shear (longitudinal flow) is enfeebled and the deformation of simple shear (the flow with a transverse velocity gradient) is increased. Since for an elastic fluid the dissipation of energy in a longitudinal field is higher than in a transverse field, this conclusion must be considered as a corollary of Helmholtz's theorem on the minimum of dissipation of energy on flow.

INTRODUCTION

The peculiarities of the convergent flow of polymeric solutions and melts are most pronounced on their squeezing through a short capillary or slit, a process characteristic for the to-day technology of spinning of polymeric fibers and films. Though the studies on the subject are very aboundant, our ideas about the convergent flow are still far from being complete (BIRD, 1977). The main feature of the convergent flow of an elastic liquid is an abrupt increase of the resistance to the flow on increasing overfall of pressure (SUL'ZHENKO, 1967) Δp. This phenomenon is reflected in the increase of the product $\Delta p.t$ which has the dimensionality of viscosity (t being the time of flow of a given volume of the liquid through the capillary). The increase of $\Delta p.t$ together with the known fact of the increase of longitudinal viscosity of elastic liquids on increase of the velocity gradient (LODGE 1964) led to the conclusion that in a convergent stream dominates the longitudinal flow. A method was even recommended for measurement of the longitudinal viscosity with aid of short capillaries (COGSWELL, 1972).

The increase of the resistance to the flow is accompanied by a formation of a birefringent inlet stream (see fig.1). The inundate inlet stream is surrounded by the so-called circulation zone the velocity of motion of the liquid within which is insignificant. The angular width of the inlet stream 2α decreases on increasing Δp, and its birefringence increase.

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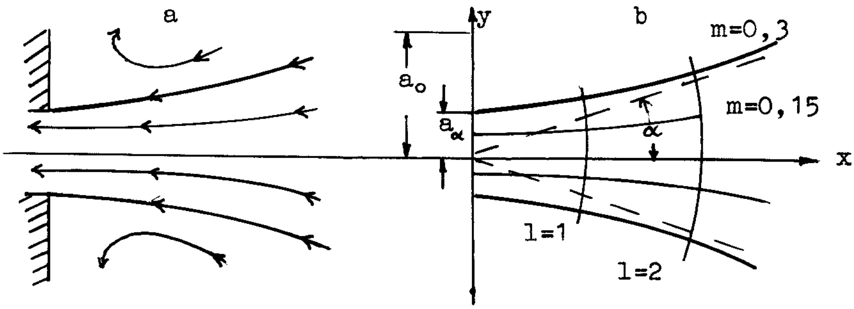

Fig.1. a).The movement of an elastic liquid on the inlet of the slit. The inundate inlet stream is shown with fat lines. b).Co-ordinate systems x, y, z and l,m,z, The dashed lines show the asymptotes of the hyperbolae which are the boundaries of the inlet stream.

RESULTS AND DISCUSSION

In this paper we shall show that though the very formation of the inlet stream is due to the increase in the longitudinal viscosity on increasing velocity gradient, the flow field within the stream is not longitudinal but mainly transverse. Higher is the resistance to the flow closer is the flow field to a transverse one.

Consider the motion in the inlet zone of a slit of a imcompressible elastic fluid with a simplest memory function (LODGE, 1964):

$$\mu = (\eta / \tau_1^2) \exp (-t/ \tau_1) \tag{1}$$

t being the time and τ_1 the relaxation time. Such a liquid has a constant viscosity η in case of an established flow in a transverse field, and in case of a longitudinal flow its viscosity increases with increasing velocity gradient. We assume:

1. The flow lines are cofocal hyperbolae whose asymptotes intersect on the z-axis of x, y, z coordinates (fig.1).
2. The velocity of flow within the circulation zone equals, at first approximation, zero.
3. The overfall of pressure on the boundary of the inlet stream in the direction normal to this boundary is also zero.

The assumption (1) and (2) are based on our own observations of the convergent flow of elastic liquids. The assumption (3) can be deduced from Bernulli's equation.

The motion along the hyperbolic lines of flow is readily presented in co-ordinates of an elliptic cylinder (MADELUNG, 1957) l, m, z which are connected with the coordinates x,y,z by the following relations:

$$\begin{aligned} x &= a_o \sqrt{1-m^2} \cdot l \ ; \quad /m/ \leqslant m_{max} \\ z &= z \\ y &= a_o \cdot m \sqrt{1+l^2} \ ; \quad 0 \leqslant l < \infty \end{aligned} \tag{2}$$

$a_o = \frac{a_\alpha}{\sin \alpha}$ determining the position of the focus on the y-axis, $2a_\alpha$ being the slit width, and $m_{max} = \sin \alpha$.

The coefficients of Lame for the chosen system of coordinates are determined from the relations:

$$H_l = \frac{a_o\sqrt{1+l^2-m^2}}{\sqrt{1+l^2}} \; ; \quad H_m = \frac{a_o\sqrt{1+l^2-m^2}}{\sqrt{1-m^2}} \; ; \quad H_z=1 \qquad (3)$$

Using the equation of imcompressibility (LOYTZANSKIY, 1978)

$$\operatorname{div} \vec{V} \cong \frac{\partial}{\partial l}(V_l H_m H_z) + \frac{\partial}{\partial m}(V_m H_z H_l) + \frac{\partial}{\partial z}(V_z H_l H_m) \qquad (4)$$

one can represent the modulus of the vector of the velocity of motion along cofocal hyperbolae in the form of series:

$$/\vec{V}/ = V_l = \sum \frac{A_i m^{2i}}{\sqrt{1+l^2-m^2}} \; ; \quad V_m = V_z = 0 \qquad (5)$$

V_l, V_m, V_z - being the components of $\vec{V}$ in coordinates l,m,z. On flow of a Newtonian liquid in a gap between hyperbolic cylinders only A_o and A_1 have nonzero values (BOYKO, 1975). In the case of an elastic liquid with a flow field limited by hyperbolic surfaces with $m_1 = \sin\alpha$ and $m_2 = -\sin\alpha$, A_2 also obtains a nonzero value. To determine A_o, A_1 and A_2 we shall use the equation:

$$Q = 2\int_o^{m_{max}} V_l H_m dm \qquad (6)$$

which expresses the dependence of the expenditure per second, Q, on a unit length of the slit on the velocity, as well as the assumptions (2) and (3):

$$V_l = 0 \qquad \text{at} \qquad /m/ = m_{max} \qquad (7)$$

$$\frac{\partial p}{\partial m} = 0 \qquad \text{at} \qquad /m/ = m_{max} \qquad (8)$$

At low velocities and Δp the viscosity of an elastic liquid is a weak function of the velocity gradient. Hence for an estimation of the pressure overfall near the boundary of the inlet stream one can use the solution for the convergent flow of an non-inertial Newtonian liquid (BOYKO, 1975):

$$\frac{\partial p}{\partial m} = \frac{2\nu l\,(1+l^2)^{1/2}}{a_o(1+l^2-m^2)^{3/2}} \cdot \left(\frac{\partial V_l}{\partial m} - \frac{mV_l}{1+l^2-m^2} \right) \qquad (9)$$

A jointed solution of eqs.(6), (7) and (8) gives a following expression for the flow field of an elastic fluid outside the slit:

$$/\vec{V}/ = V_l = A\,\frac{(\sin^2\alpha - m^2)^2}{\sqrt{1+l^2-m^2}} \; ; \quad A = \frac{-4Q}{a_o(2\alpha + \alpha\cos 4\alpha - 0{,}75\sin 4\alpha)} \qquad (10)$$

This leads in a simple way to equations connecting the velocity of rotation ω of an element of volume of the medium and the quadratic invariant T_2 of the tensor of rates of deformation with the expenditure per second and the angular width of the inlet stream 2α :

$$T_2 = -\dot{\gamma}_{11}\cdot\dot{\gamma}_{mm} + \dot{\gamma}^2_{1m} \tag{11}$$

$$\omega = \frac{/\mathrm{rot}_z \vec{V}/}{2} = \frac{1}{2H_1H_m}\cdot\frac{\partial}{\partial m}(V_1H_1) = -\frac{2Am\sqrt{1-m^2}\,.(\sin^2\alpha - m^2)}{a_o(1+1^2-m^2)} \tag{12}$$

$\dot{\gamma}$ being different from zero components of the deformation rates tensor:

$$\dot{\gamma}_{1m} = \frac{1}{2}\,\frac{H_1}{H_m}\cdot\frac{\partial}{\partial m}\left(\frac{V_1}{H_1}\right) = \frac{-Am\sqrt{1-m^2}(\sin^2\alpha - m^2)[2(1+1^2)-(\sin^2\alpha + m^2)]}{a_o(1 + 1^2 - m^2)^2} \tag{13}$$

$$\dot{\gamma}_{11} = -\dot{\gamma}_{mm} = \frac{1}{H_1}\cdot\frac{\partial V_1}{\partial 1} = \frac{-A}{a_o}(\sin^2\alpha - m^2)^2\,\frac{1\sqrt{1+1^2}}{(1+1^2-m^2)^2} \tag{14}$$

To estimate the character of the flow field it is necessary to find the ratio $\omega/\sqrt{T_2}$ keeping in mind that $\omega/\sqrt{T_2}=1$ corresponds to a flow with a transvers velocity gradient (deformation of simple shear), and $\omega/\sqrt{T_2}=0$ corresponds to a flow with a longitudinal velocity gradient (deformation of pure shear).

The hydrodynamic field outside the slit is nonestablished, which leads to the variability of $\omega/\sqrt{T_2}$ for a moving element of volume of the medium.

We shall estimate the character of the flow field by means of averaging $\omega/\sqrt{T_2}$ along a flow line. This is simplified by introduction of reduced Cartesian coordinates: $X=\frac{x}{a_\alpha}$, $Y = \frac{y}{a_\alpha}$, $Z = \frac{z}{a_\alpha}$ where Y attains the values +1 and -1 on the edges of the slit. On the y-axis 1=0, hence in correspondence with equations (2):

$$Y = \frac{m}{\sin\alpha} \quad \text{at} \quad 1=0 \;; \quad \left(\frac{dm}{dY}\right)_{1=0} = \sin\alpha \tag{15}$$

Let us calculate the average values of ω^2 and T_2 isside a flow tube limited by two hyperbolic surfaces, the first intesecting the Y-axis at Y=Y, and the second at Y=Y+dY. The corresponding quantities are:

$$\langle\omega\rangle = \frac{dm\int_0^\infty \omega^2 H_1H_m d1}{dY} = \sin\alpha\int_0^\infty \omega^2 H_1H_m d1 \tag{16}$$

$$\langle T_2\rangle = \sin\alpha\int_0^\infty T_2H_1H_m d1 \tag{17}$$

Using eqs. (11) - (14) we obtain finally:

$$\langle \omega^2 \rangle = 4A^2 \sin\alpha \,.\, m[\sin^2\alpha - m^2]^2 \arcsin m \qquad (18)$$

$$\langle T_2 \rangle = \frac{A^2}{2} \sin\alpha (\sin^2\alpha - m^2)^2 \left\{ (\sin^2\alpha + m^2)^2 \Phi_1 + m^4 \Phi_2 \left(\frac{\cos^2\alpha}{1-m^2} + 1\right)^2 \right\}$$

$$\Phi_1 = \frac{\arcsin m}{m^3} - \frac{\sqrt{1-m^2}}{m^2} \; ; \quad \Phi_2 = \frac{\arcsin m}{m^3} + \frac{\sqrt{1-m^2}}{m^2} \qquad (19)$$

The results of calculations based on (18) and $(19)_2$ for $\alpha = 30°$ and 14,5° are given in the table. The data for $\langle \omega^2 \rangle$, $\langle T_2 \rangle$, $\langle \omega^2 \rangle^{1/2} / \langle T_2 \rangle^{1/2}$ for a Newtonian liquid ($\alpha = 90°$) pouring into the slit are also presented. In this case according to (BOYKO, 1975):

$$/\vec{V}/ = V_1 = - \frac{2Q}{a_o \cdot \pi} \cdot \frac{1 - m^2}{\sqrt{1 + l^2 - m^2}}$$

The data of the table show that a purely longitudinal flow occurs only at Y=0. The range of Y where the longitudinal flow is predominant ($\langle \omega^2 \rangle^{1/2} / \langle T_2 \rangle^{1/2} < 0,5$) narrows with decreasing α. The decrease of α is followed by a decrease of the deformation rate at Y=0 (from 0,135 and 0,141 for $\alpha = 90°$ and $\alpha = 30°$ down to 0,07 for $\alpha = 14,5°$).

Simultaneously a very strong increase of the deformation rate occurs in the vicinity of Y=0,6 where, as it follows from the ratio ($\langle \omega^2 \rangle^{1/2} / \langle T_2 \rangle^{1/2}$), a flow with a transverse velocity gradient occurs (from 0,153 for $\alpha = 90°$ up to 2,02 for $\alpha = 14,5°$). The range of α corresponding to $\langle \omega^2 \rangle^{1/2} / \langle T_2 \rangle^{1/2} \approx 1$ widens on decrease of α. A sharp increase of the rate of simple shear deformation at decreasing α explains the increase of the product $\Delta p . t$ on increasing Δp. Approximately equal values of $\langle T_2 \rangle$ in the vicinity of Y=0 at $\alpha = 90°$ and 30° explain why the angular width the streams observed in polymeric liquids never exceeds 60°.

Thus the occurence of a birefringent inlet stream and the decrease of its angular width can be considered as a process of such a transform of the flow field where the deformation of pure shear is enfeebled, and the deformation of simple shear is sharply enforced. A substantial difference of the hydrodynamic field inside the inlet stream from longitudinal one is one of the causes of the fact that in devices with a single orifice there occurs no effective uncoiling of macromolecular chains before the entrance in the orifice.

TABLE The parameter of the hydrodynamic field of different Y.

Y	$\langle\omega^2\rangle (a_\alpha/Q)^2$			$\langle T_2\rangle (a_\alpha/Q)^2$			$\langle\omega^2\rangle^{1/2}/\langle T_2\rangle^{1/2}$		
	90°	30°	14,5°	90°	30°	14,5°	90°	30°	14,5°
1	2	3	4	5	6	7	8	9	10
0	0	0	0	0,135	0,141	0,07	0	0	0
0,1	0,004	0,066	0,14	0,135	0,192	0,20	0,17	0,59	0,83
0,2	0,016	0,250	0,51	0,137	0,333	0,56	0,35	0,87	0,96
0,3	0,037	0,507	1,04	0,136	0,532	1,05	0,52	0,58	1,00
0,4	0,067	0,770	1,58	0,143	0,737	1,56	0,68	1,02	1,01
0,5	0,106	0,962	1,97	0,147	0,889	1,93	0,85	1,04	1,01
0,6	0,156	1,01	2,06	0,153	0,928	2,02	1,01	1,04	1,01
0,7	0,220	0,882	1,79	0,163	0,811	1,75	1,16	1,04	1,01
0,8	0,301	0,578	1,16	0,177	0,541	1,15	1,30	3,03	1,00
0,9	0,408	0,205	0,41	0,202	0,198	0,41	1,42	1,02	1,00
1,0	0,637	0	0	0,318	0	0	1,41	1,00	1,00

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